Dynamic frequency estimation under low frequency AM/PM modulation

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Abstract: This work proposes the utilization of an iterative algorithm applied to some fitness functions to estimate accurately the frequency of a sampled signal with combined amplitude and phase modulations. Such modelling was implemented in Labview environment to simulate PMU dynamic measurements. The errors obtained with a few cicles are low enough to allow the utilization of such method in PMU calibration systems.

Keywords: frequency estimation, PMU, smart grid

1. INTRODUCTION

The PMUs are being increasingly used in the grid control systems to monitor the dynamic performance of the electric grid.

Much effort is being placed in the world to develop metrological infrastructure to provide traceability to national measurement standards for PMUs [1-3].

The IEEE Standard C37.118 [4] defines a measurement bandwidth test to be applied, with combined phase and amplitude modulations. Dynamic frequency measurements are a key concern then. However, an amendment published in 2014 [5] no longer requires the test with combined modulations, but yet they shall be done separatedly.

A crucial element of any PMU is the mathematical algorithm for calculation of the phasors. Least squares fitting can be used to extract the waveform parameters from the samples [6]. This paper proposes the utilization of the Taylor Expansion Method, described in [7], together with the Levenberg-Marquardt algorithm [8,9] to estimate the phasor frequency under low phase and amplitude variations. The algorithms were tested in a PMU simulation software developed in Labview environment.

2. MODULATED SIGNAL

2.1. Generated signal

The amplitude and phase modulation shall be used to determine the synchrophasor measurement bandwidth, in the form of the equation (1).

$$X_1 = X_m[1 + k_x \cos(\omega t)]\cos[\omega_0 t + k_a \cos(\omega t - \pi)] \quad (1)$$

The dynamic frequency and the frequency error of the phasor is

$$f(nT) = \omega_0/2\pi - k_a \sin(\omega nT - \pi) \quad (2)$$

$$FE == |f_{true} - f_{measured}|, \quad (3)$$

where nT is the reporting time and ω is the modulation frequency [4].

2.2. Taylor Method

The modulated signals can be expanded in Taylor series for the frequency estimation,

$$(v_0 + v_1 t + v_2 t^2 + \cdots)\sin\big((f_0 + f_1 t + f_2 t^2)t + \theta\big) \quad (4)$$

The order of the model can be expanded to obtain better approximations. Such a method has succeeded under low modulation indexes, with a time window of about 3 cycles [7]. When there is only amplitude modulation, the frequency parameters can be reduced to order zero. The same applies when performing phase modulations, so the amplitude parameters can be reduced to only one parameter.

2.3. Levenberg-Marquardt algorithm

The Levenberg-Marquardt algorithm is a general method applied to least squares curve-fitting problems. Given a set of m empirical pairs of independent and dependent variables, (xi, yi), it optimizes the parameters $\boldsymbol{\beta}$ of the model curve $f(x,\beta)$ so that the sum of the squares of the deviations

$$S(\beta) = \sum_{i=1}^{m} [y_i - f(x_i, \beta)]^2$$

be minimal.

In each iteration step, the parameter vector, $\boldsymbol{\beta}$, is replaced by a new estimate, $\boldsymbol{\beta}+\boldsymbol{\delta}$, until the difference in S between two iterations or the step $\boldsymbol{\delta}$ fall below predefined limits. The parameter updates between the gradient descent update and the Gauss-Newton update are adapted. More detail can be found in [8,9].

Care must be taken when applying this model if the input matrix may not be singular, unless the algorithm will not succeed. It is also very sensitive to the initial guess of the parameters.

3. LABVIEW IMPLEMENTATION

A simulated PMU calibrating system is implemented in Labview environment, taking the theoretical frequency as a reference. A dynamic frequency and amplitude generator simulates the input signals to the system.

A Levenberg-Marquardt algorithm was also implemented, to estimate the parameters of the Taylor expansion models of orders 1 and 2.

The software allows the user to set the parameters of the generated signal function, the number of cicles simulated, the size of the sampling window and the sampling frequency.

There were performed tests with modulation index of 0,1, modulation frequencies ranging from 0,1 Hz to 2,0 Hz, frequency of the carrier set at 60Hz, 3 cycles of observation window, over at least two entire modulation cycles, at a sampling rate of 1200 Hz. The initial guess used is the *a priori* information about the known theoretical value of frequency, k_x, k_a and phase and slow values such as 0,001 to the other parameters. At each time stamp, the frequency error is recorded and the iteration process starts again to update the parameters.

4. SIMULATION RESULTS

A comparison of the maximum frequency errors obtained with different modulation signals is shown in the figures 1 and 2. The figures 3 and 4 show the performance of the 1st and 2nd order Taylor models. Figures 5 to 10 show the dispersion of the data for each simulation.

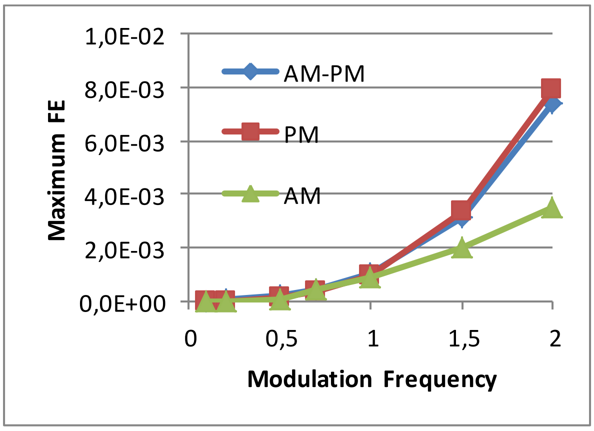

Figure 1 – Maximum FE - Taylor 1st order method

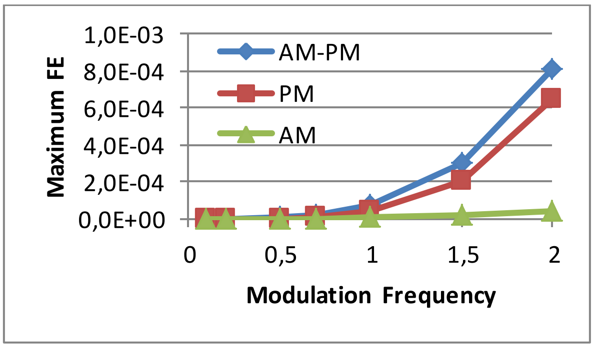

Figure 2 – Maximum FE comparison - Taylor 2nd order method

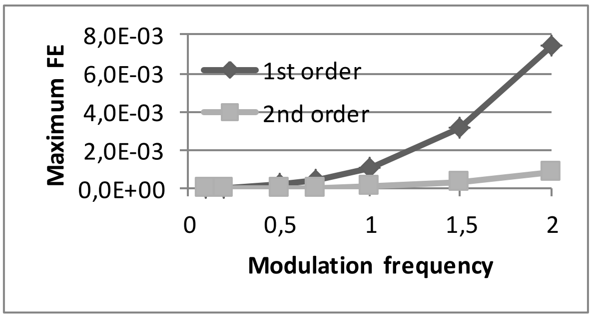

Figure 3 – Maximum FE with AM-PM combined

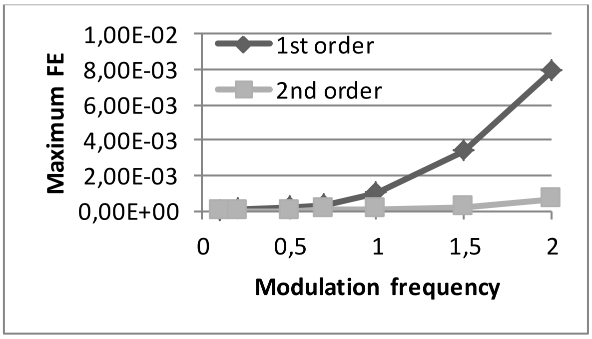

Figure 4 – Maximum FE with PM only

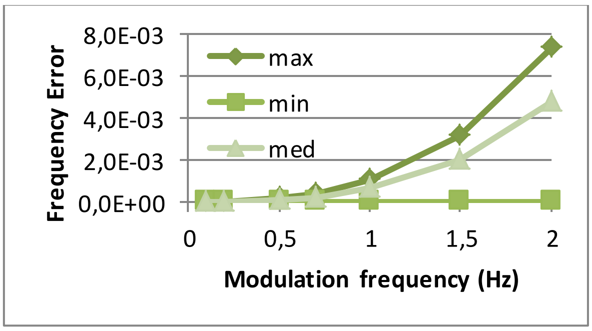

Figure 5 – FE with combined AM-PM – Taylor 1st order method

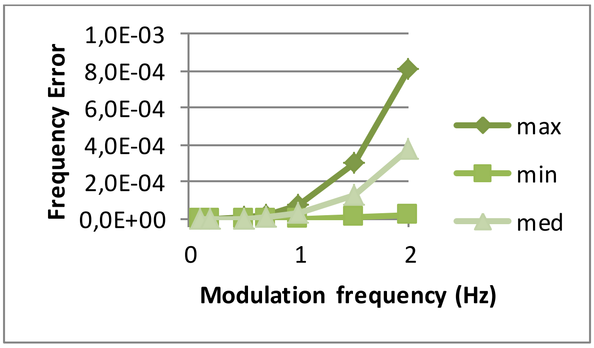

Figure 6 - FE with combined AM-PM – Taylor 2nd order method

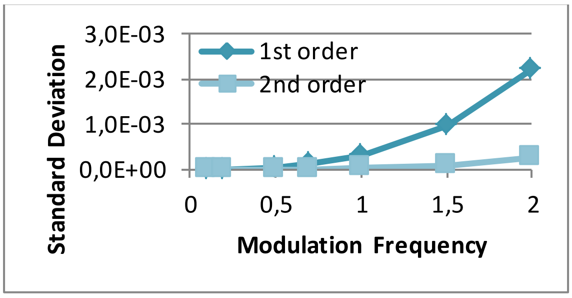

Figure 7 – Standard Deviation with combined AM-PM

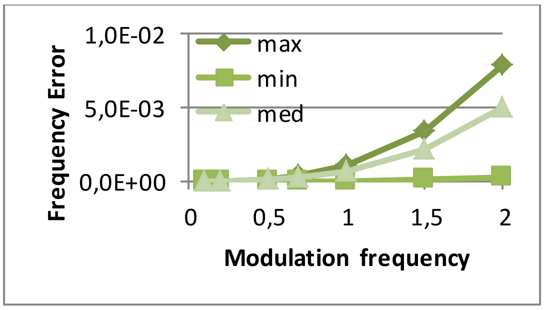

Figure 8 - FE with PM only - Taylor 1st order method

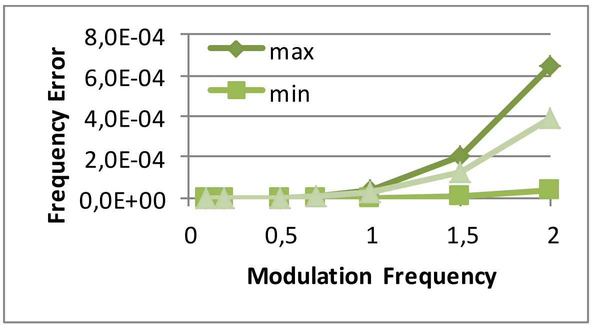

Figure 9 - FE with PM only - Taylor 2nd order method

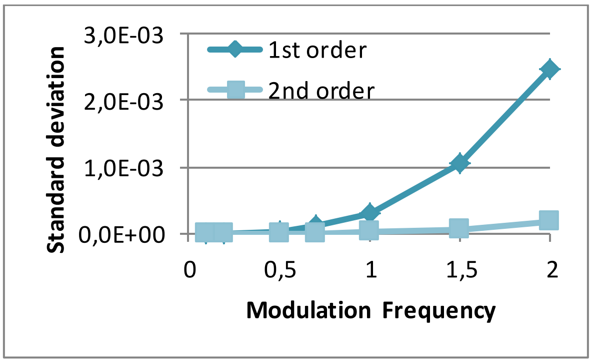

Figure 10 – Standard Deviation – PM only

5. CONCLUSION

The results indicate that the second order model gives the least maximum frequency errors as well as the least dispersion during the modulation cycle. Both models show worse results as the modulation frequency gets higher.

The proposed method gives a quite good approximation of the frequency and therefore can be used to develop a PMU reference system for dynamic frequency tests. It has the advantage of using just a few cicles and a low sampling frequency to estimate the parameters of the model.

In a realization of such a system, the uncertainties associated with the numerical method must then be combined with the sampling uncertainties and those associated with the attenuators used to accommodate real signals.

6. REFERENCES

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