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AC resistance measurements using digital sampling techniques

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Abstract: Here we will present measurement results for ac resistors calibrations, made with a four terminal-pair digital impedance bridge, developed at Inmetro. This bridge operates in a wide frequency and value ranges, for both in-phase and quadrature measurements, with uncertainties bearing some μΩ/Ω.

Keywords: ac resistance, digital sampling, impedance measurements

1. INTRODUCTION

In the last few years, Inmetro has been receiving requests to calibrate standard resistors in alternating current (ac), from primary and secondary laboratories, industries, electric power companies, research centers, and universities. At the present moment, the calibration of resistors ac is being performed by a substitution method, however, this method presents several drawbacks, such as limitation of the calibration scope, current and frequency ranges, poor repeatability, high uncertainty, and dependency of standards calibrated at other NMIs.

To address these difficulties, the Electrical Standardization Metrology Laboratory (Lampe) is developing two projects in parallel. The primary system, which aims at tracing the capacitance unit to the quantum Hall effect, in consequence, granting the traceability of ac resistance [1-2]. The second project aims at developing a digital bridge for impedance measurements [3].

In this paper, we will present results for ac resistors measurements performed with the digital bridge for impedance measurements. This bridge can be applied to comparisons of four-terminal pair impedances of the same quantity (in-phase) and of different nominal values, i.e., pairs of ac resistors or capacitors standards; or for comparisons in quadrature, i.e., ac resistors with capacitors. This system can also be applied to a range of impedance measurement tasks, such as determining the impedance standards secondary parameters. The digital bridge is able to operate in a wide frequency range (from a few Hz to 3 kHz) and impedance values (10 Ω to 100 kΩ), with measurement uncertainties bearing some μΩ/Ω.

In the following section, we will briefly describe the principle of the bridge operation. In Sections 3 and 4 we will present ac resistance measurements performed with the digital bridge with a preliminary uncertainty analysis.

2. DIGITAL BRIDGE FOR IMPEDANCE MEASUREMENTS

The digital bridge proposed in [3] employs digital synthesizers that operate synchronously with the internal reference (clock) of a digitizer. The amplitude and the phase of the output signals are

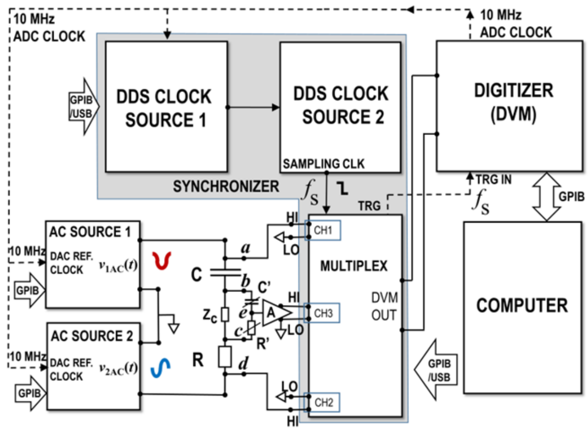

Figure 1 - Digital Bridge Diagram

finely adjusted by software, buffered by specially designed amplifiers. Adaptive digital regulators (in the control software) [4], allow the balance of the bridge. Hardware and software aspects of the digital bridge are detailed in [5].

2.1. Digital System

The system shown in figure 1 operates as a voltage comparator [6]. Voltages of the sinusoidal signal generators connected in series, $v_{1AC}(t)$ and $v_{2AC}(t)$ are adjusted in-phase and amplitude by software, resulting in the total voltage measured between points *a* and *d* with respect to ground, and with respect to *e*.

The control software then calculates the phase and amplitude of the generators, which would nullify the voltage at point *e* (see figure 1), i.e., from the Kelvin arm composed by C' and R', which are used to compensate for voltage drops in the connection between the impedances under comparison, represented by the Z_c impedance. The Kelvin arm must satisfy the following relation R'C' = RC. The bridge is powered by amplifiers, as described below; more detail can be found in [5].

2.2. Amplifiers and Synchronization Circuit

The digital bridge has some important amplifiers in the circuit. We can highlight the "Kelvin" amplifier, which is a unity gain buffer with the task of promoting the impedance matching, and allowing the voltage at the point *e* of the circuit to be sampled without loading the Kelvin arm with the input impedance of the digitizer's analog-to-digital converter (ADC).

On the other hand, the high-gain tracking amplifier (not shown in figure 1), feeds the residual voltage from point *e* in series with the output of the AC SOURCE 1. Thus inherent amplitude variations of sources 1 and 2 can be balanced out, significantly increasing the stability of the voltage ratio measurements between the *ab* and *cd* terminals. This amplifier allows the bridge to attain greater performance and higher repeatability even when compared with more complex projects [6-7].

The use of a low-noise, high-gain integrating circuit for the tracking amplifier ensures stability, allowing many different impedance values to be compared. In addition, the tracking amplifier relaxes stringent requirements on voltage sources that could be used in the system. Therefore, relatively simple and inexpensive voltage generators can be employed in the digital bridge.

The gray shaded area in figure 1 is a modern synchronizer, based on direct digital synthesizers (DDS) [4,8] to control the coherent acquisition of a number of samples from each channel.

3. IMPEDANCE MEASUREMENTS

In order to validate the digital bridge, we made several measurements with multiple four-terminal pairs impedance standards, previously calibrated with coaxial bridges [1-2]: a set of ac resistors with values between $100\,\Omega$ and $10\,k\Omega$ and a $10\,nF$ capacitor. The secondary parameters, i.e., the ac resistors' time constant τ and the capacitor dissipation factor $\tan\delta$ were also known.

3.1. In-Phase ac Resistance Measurements

Equations (1) and (2) show a simplified model for estimating the parameters of the resistor R_X

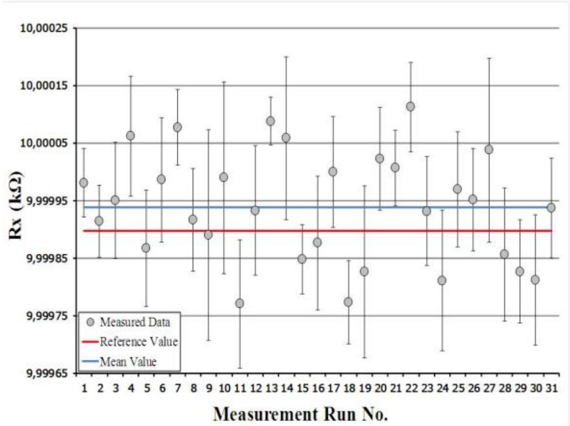

Figure 2 - In-Phase ac Resistance Measurement

using a known ac resistance standard $Z_S = R_S(1 + j\omega\tau_S)$:

$$R_X = R_S(A - \omega\tau_S B) \qquad (1)$$

$$\tau_X = \frac{B + A\omega\tau_S}{\omega(A - B\omega\tau_S)} \qquad (2),$$

where $A + jB$ is the complex ratio of the fast Fourier transform of the measured voltages (A is the real part and B is the imaginary part), and ω is the fundamental frequency.

The digital bridge allows impedance comparisons with different nominal values, with a gain ratio of up to 10. So we have chosen a standard resistor of $1\ k\Omega$, previously calibrated with a measurement uncertainty U = $0.7\ \mu\Omega/\Omega$, as the reference standard R_S. Using this standard we calibrated a $100\ \Omega$ and a $10\ k\Omega$ resistors, both previously known at the frequency of 1 kHz.

Table 1 shows the ac resistance values measured with the digital bridge (M_{eas}) in comparison with the calibration results (reference value - R_{ef}). Each resistance value measured with the digital bridge corresponds to a set of at least 20 measuring runs, where each run consists of eight determinations by digital sampling.

We can observe that the differences (Δ) between the values measured with the digital bridge in comparison with the reference calibration values [1-2] are less than $5\ \mu\Omega/\Omega$, even with the digital bridge operating with the maximum gain of 10.

Table 1: ac resistance measurements.

R_X	M_{eas} (Ω/kΩ)	R_{ef} (Ω/kΩ)	Δ ($\mu\Omega/\Omega$)
$100\ \Omega$	100,0009	100,0006	3,2
$10\ k\Omega$	9,999938	9,999897	4,1

Figure 2 shows the digital bridge measurement results at 1 kHz, for the calibration of $R_X = 10\ k\Omega$ using the $R_S = 1\ k\Omega$ as the reference standard (see table 1, line 2). For this calibration we had a set of 32 runs. The figure also shows the bars to indicate the repeatability for each run, the mean value for the 32 runs, and the reference value for R_X, obtained with the primary system coaxial bridge.

Several additional measurements have shown that the digital bridge is able to operate at frequencies between 400 Hz and 1.6 kHz, for resistance values from $10\ \Omega$ to $100\ k\Omega$, with similar or better results than those shown in Table 1.

3.2. Quadrature ac Resistance Measurements

In addition to performing in-phase measurements, i.e., comparing two standards of the same magnitude, the digital bridge also perform impedance comparisons in quadrature, i.e., the comparison of an ac resistor with a capacitor. To once again demonstrate the effectiveness of the digital bridge, we have made a second sequence of measurements, the calibration of a $10\ k\Omega$ ac resistorR_X, using two different reference standards: 1) with the $1\ k\Omega$ ac standard resistor, described in subsection 3.1; 2) with a $10\ n$F standard capacitor. Equations (3) and (4) show the model to estimate the parameters of R_X from a known capacitance standard $Z_S = ({}^{1}/_{\omega C_S})(\tan\delta + j)$:

$$R_X = \frac{1}{\omega C(B + \omega\tau A)} \qquad (3)$$

$$\omega\tau = \frac{\tan\delta + (A/B)}{1-\tan\delta (A/B)} \quad (4)$$

Table 2 presents the values for R_X e τ_X measured with the digital bridge when using the reference standards: the 1 $k\Omega$ resistor (in-phase) and the 10 nF capacitor (quadrature). Both measurements were made at the frequency of 1.592 kHz and show nearly identical results.

Table 2: Measurements of R_X - In-Phase x Quadrature

Z_X	In-Phase	Quadrature	$\Delta\ (\mu\Omega/\Omega)$
$R_X\ (k\Omega)$	10,000193	10,000192	0,2
$\tau_X\ (ns)$	13	14	----

4. UNCERTAINITY CALCULATIONS

In this section we will present an uncertainty evaluation for the calibration of R_X, at 1.592 kHz, using the 1 $k\Omega$ ac resistor as the reference standard (in-phase measurement). Table 3 shows the main uncertainty sources.

The main uncertainty contributions arise from digitizer noise, quantization and the gain correction (variations). The latter contribution becomes more pronounced when measuring resistors with different nominal values. Furthermore, amplitude instabilities of the voltage sources impair substantially measurements, with are partially counteracted by the tracking amplifier. Other uncertainty components may be attributed to inevitable electromagnetic coupling in the internal circuitry of the bridge [3].

We can observe in Table 3 that the system shows a combined uncertainty of 4.0 $\mu\Omega/\Omega$. This uncertainty may significantly be reduced to some parts 10^{-7} if the currently used voltage sources are replaced by others with higher amplitude and phase stability. These are presently under development.

Table 3: R_X Measurements - Uncertainty

Source	u (μΩ/Ω)	Type
Digitizer	2,0	B
Amplitude Instability of the Voltage Sources	3,0	B
Reference Standard	0,4	B
Repeatability (on 64 cycles)	1,5	A
Combined	4,0	

Moreover, using current amplifiers and some modifications in the circuit, we can employ the digital bridge for the measurement ac current shunts. This work is still in progress.

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